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DESIGN OF HELICAL TEETH FOR BENDING (K RASCHETU KOSYKH ZUBEV NA--ETC(U)
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by

T.V. Dolgova

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ROYAL AIRCRAFT ESTABLISHMENT

14 RAE - Library Translation 1925

Received for printing May 1977

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DESIGN OF HELICAL TEETH FOR BENDING

(K RASCHETU KOSYKH ZUB'EV NA IZGIB)

by

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Izv. Vyssh. Uchebn. Zaved, Mashinostroenie 11, 37-40 (1974)

Translator
J.W. Palmer

12 10p.

Translation editor
D.K. Brighton

AUTHOR'S SUMMARY

A solution is given to the problem of selecting the shape coefficient of a helical gear tooth and the accuracy of existing calculations for helical gears.

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The design of helical gear teeth normally involves the determination of the shape coefficient obtained from the reduced or equivalent spur gear having $z_e = z/\cos^3\beta$ teeth, where β is the helix angle. It is assumed that the shape of the helical tooth in the normal plane corresponds to the shape of the reduced spur gear tooth. Coincidence of the radii of curvature of the equivalent gear tooth and the normal section of the helical gear tooth in the plane of meshing makes it possible to consider the shape of these teeth equivalent solely at the point under investigation. The correspondence of the shape of the teeth away from the point in the root region has been little investigated. However, replacement of a helical gear tooth by an equivalent one could introduce a considerable error into the calculated shape of the tooth at the place which is most important for its fracture strength. In this connection V.A. Andozhskii suggests that tooth bending calculations should take their true shape into account, and that replacement of a helical gear by a conventional spur gear is inadequately substantiated, since, in his view, the magnitude of the stresses is considerably distorted.

The investigations we performed had as their objective the determination of the geometry of a helical tooth in a normal section and by theory of elasticity methods to find the maximum stresses in it, which would make it possible to solve for the correctness and accuracy of the existing recommendations for choice of the shape coefficient for helical gear teeth.

The method of 'normal planes' developed by V.D. Andozhskii was adopted to determine the geometry of a helical tooth. The normal plane passes through the vector of the load P_N applied to the apex of the tooth, and the axis of symmetry of the profile. The profile of the tooth in the normal plane is symmetrical. The corresponding profiles are shown in Fig 1, while here the profile of the equivalent gear is shown.

The bending stresses of the helical tooth were determined from the formulae for plane theory of elasticity by conformal mapping of the contour of the tooth on to a semi-plane[2].

The mapping function assumes the form:

$$Z = \omega(W) = W + \sum_{n=1}^{5} \frac{a_n}{W - ib_n} + iy_0 \quad *$$

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* Calculations were performed using seven coefficients a_n, b_n of the mapping function, but the accuracy of the calculations was only marginally improved.

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where Z is the complex variable on the plane of the projection, W is the complex variable on the mapped plane, y_0, a_n, b_n are the coefficients of the mapping function.

The calculations were performed for two tooth numbers $z = 17$ and $z = 30$ with varying tooth helix angle β from 10 to 40°, with module m_N, tooth height coefficient $f = 1$, centre distance $c = 0.25$ m and a load $P_N = 1$ kg/mm.

The results of the calculations are collected in Tables 1-2 and shown in Fig 2.

From analysis of the data the calculated maximum stresses σ^P_{max}, determined in the normal plane of a helical tooth, do not differ practically from the stresses in the tooth of an equivalent spur gear, both for $z = 17$ and $z = 30$. The maximum difference σ^P_{max} amounts to ~2% which lies within the limits of error due to inaccuracies in the calculated shape of the tooth-shaped projection: the deviation of the radii of curvature of the tooth and of the projection mapping it (shown in Fig 1 by a dotted line), for which the stresses were found, lie within the limits of 8.53% to 16.5%, which gives an error in stress concentration coefficients $\Delta\alpha_\sigma$, and consequently of the actual stresses $\Delta\sigma^P_{max}$ of 2.47% to 4.9% respectively. This inaccuracy may be allowed for by refining the true stresses in the root of the tooth. The divergence between the true tooth stresses for the equivalent and helical gears σ_{max} varies from 1.75% to 2.84% for the corresponding difference of the radii of curvature at the critical point of 0.34%, 1.89%. During analysis of the results it was found that the relationship between the theoretical stress concentration factor α_σ and the ratio S/ρ (S is the width of the critical section, ρ is the radius of curvature at the critical point) is: $\alpha_\sigma = 1 + 0.273 \sqrt[3]{\left(\frac{S}{\rho}\right)^2}$ which was obtained for spur gears[2]. This relationship was also used to determine the concentration factor α_σ.

The good agreement between the stresses does not however indicate a precise coincidence between the shape of the profile of the teeth of an equivalent gear and the profile of a helical tooth in a normal section. It will be seen from Fig 1 that the transposed curves of the teeth differ in curvature. The maximum difference in the radii of curvature amounts to 10.51% for $z = 17$, $\beta = 40^\circ$ (Table 2). However the deviation in the curvature of the tooth roots does not give changes in the stresses ($\Delta\alpha_\sigma = 0.27\%$) which are noticeable

from a practical point of view. Clearly the increase in the angle γ (Fig 1) is equal to ~5% with the apex of the tooth of the equivalent gear, and consequently the reduction of the bending lever arm, to a certain extent compensates for the increase in the stress concentration caused by a reduction in ρ .

It is interesting to compare the results obtained with the data given in the investigation[3]. The bending stress coefficients A_m have been obtained by the methods of non-parallel sections and non-localized elliptical coordinates. They gave results fairly close to the true shape of the tooth in the normal plane. The coefficients obtained using one of these methods are given in Fig 2 [3]. The divergencies with these results are particularly noticeable with small numbers of teeth. At a helical tooth helix angle $\beta = 10^o$ the divergence in the stresses $(1/A_m)$ ~5% for $z = 17$ and $z = 30$. With an increase of angle β to 40^o the difference in the stresses increases to ~40%.

The calculations enable the conclusion to be drawn that in spite of the discrepancy in the profiles between the equivalent and helical teeth in the normal plane, it is possible, with an accuracy adequate for engineering calculations, to replace a helical gear by an equivalent spur gear having $z_e = z/\cos^3\beta$. The shape coefficients of the two teeth in the normal plane will correspond.

Table 1

γ rad

β^o	ρ_c	ρ_v	$\frac{\rho_v - \rho_c}{\rho_c}$ %	S	α_{σ_c}	α_{σ_v}	$\frac{\alpha_{\sigma_v} - \alpha_{\sigma_c}}{\alpha_{\sigma_c}}$ %	σ^{ρ}_{max}	σ_{max}	γ rad
					$z = 17$					
10	0.56908	0.50978	-10.42	1.92122	1.61200	1.65878	+2.90	4.031	3.918	0.52846
20	0.56215	0.50938	-9.39	1.97383	1.62860	1.67129	+2.19	3.968	3.883	0.49974
30	0.55034	0.50317	-8.53	2.09432	1.66290	1.70394	+2.47	3.744	3.654	0.47098
40	0.54163	0.49337	-8.31	2.21104	1.69470	1.73957	+2.64	3.599	3.506	0.43099
					$z = 17/\cos^3\beta$					
10	0.58298	0.54485	-6.54	1.80682	1.59700	1.62478	+1.74	4.010	3.941	0.53031
20	0.56023	0.51658	-7.79	1.96792	1.62900	1.66340	+2.11	3.896	3.815	0.51205
30	0.52570	0.46801	-10.97	2.07268	1.67890	1.73358	+3.26	3.746	3.628	0.48379
40	0.48464	0.43607	-10.02	2.19336	1.74419	1.79832	+3.10	3.624	3.515	0.44896
					$z = 30$					
10	0.49452	0.41520	-16.04	2.15983	1.73331	1.81654	+4.80	3.690	3.521	0.46445
20	0.48799	0.40713	-16.57	2.20311	1.73658	1.82171	+4.90	3.644	3.474	0.44913
30	0.47908	0.40203	-16.08	2.26586	1.77760	1.86061	+4.67	3.574	3.414	0.42649
40	0.45826	0.39859	-13.02	2.33646	1.80566	1.88373	+4.32	3.500	3.355	0.40049
					$z = 30/\cos^3\beta$					
10	0.50385	0.47090	-4.75	2.13698	1.71264	1.74552	+1.39	3.672	3.621	0.46545
20	0.48903	0.42156	-13.8	2.17942	1.73710	1.81328	+4.38	3.635	3.482	0.45276
30	0.46701	0.40230	-13.86	2.23150	1.77190	1.85220	+4.53	3.571	3.416	0.43250
40	0.44156	0.37886	-14.18	2.31164	1.83120	1.90821	+4.21	3.535	3.392	0.41029

Table 2

ρ_c^e, σ_{max}^e - ARE THE RADIUS OF CURVATURE AND THE MAXIMUM BENDING STRESSES OCCURRING AT THE ROOT OF THE TOOTH OF AN EQUIVALENT GEAR

Helix angle β (deg)	Tooth number z = 17		Tooth number z = 30	
	$\frac{\rho_c^e - \rho_c}{\rho_c}$ %	$\frac{\sigma_{max}^e - \sigma_{max}}{\sigma_{max}}$ %	$\frac{\rho_c^e - \rho_c}{\rho_c}$ %	$\frac{\sigma_{max}^e - \sigma_{max}}{\sigma_{max}}$ %
10	+2.44	+0.587	+1.89	+2.84
20	-0.34	-1.75	+0.21	+0.23
30	-4.48	-0.712	-2.52	+0.06
40	-10.51	+0.257	-3.65	+1.103

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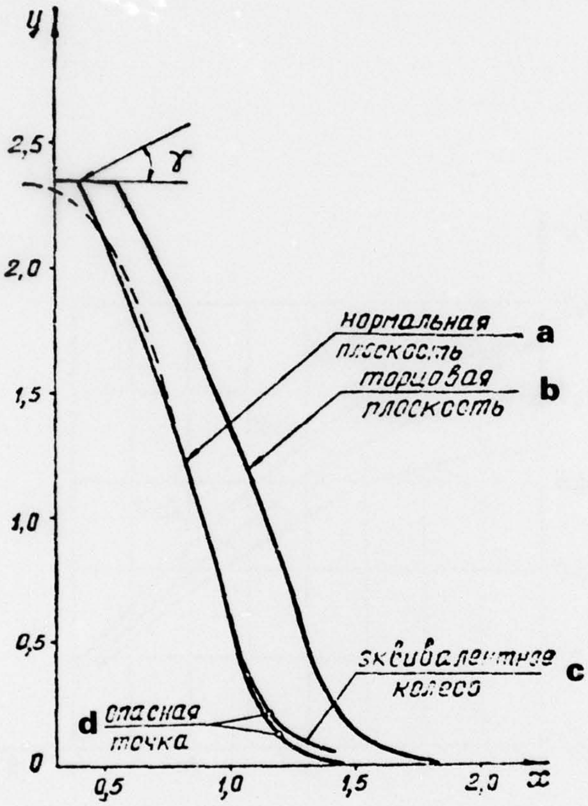

Fig 1 Profiles of helical and equivalent gears:
P_N = 1 kg/mm; z = 30; β = 40°

Key:
a = normal plane
b = transverse plane
c = equivalent gear
d = critical point

Fig 2

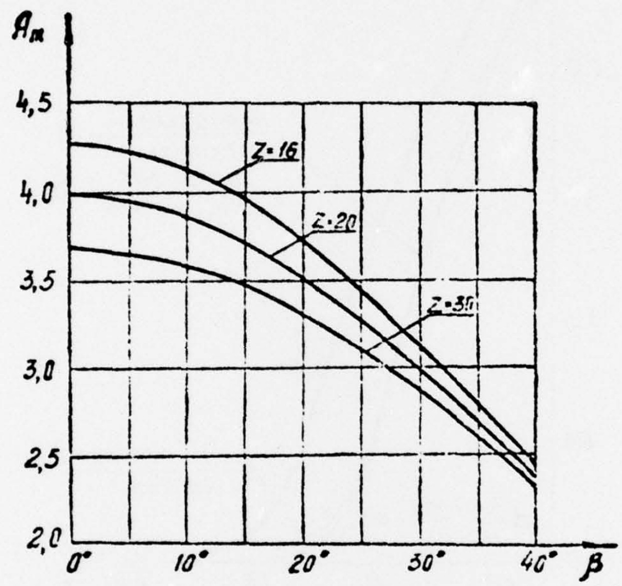

Fig 2 Relationship of A_m to β^0 for un-corrected gear teeth

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